كلية الحاسبات والذكاء الإصطناعي

# BS111

# Probability and Statistics

## Lecture 01

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# Introduce Myself

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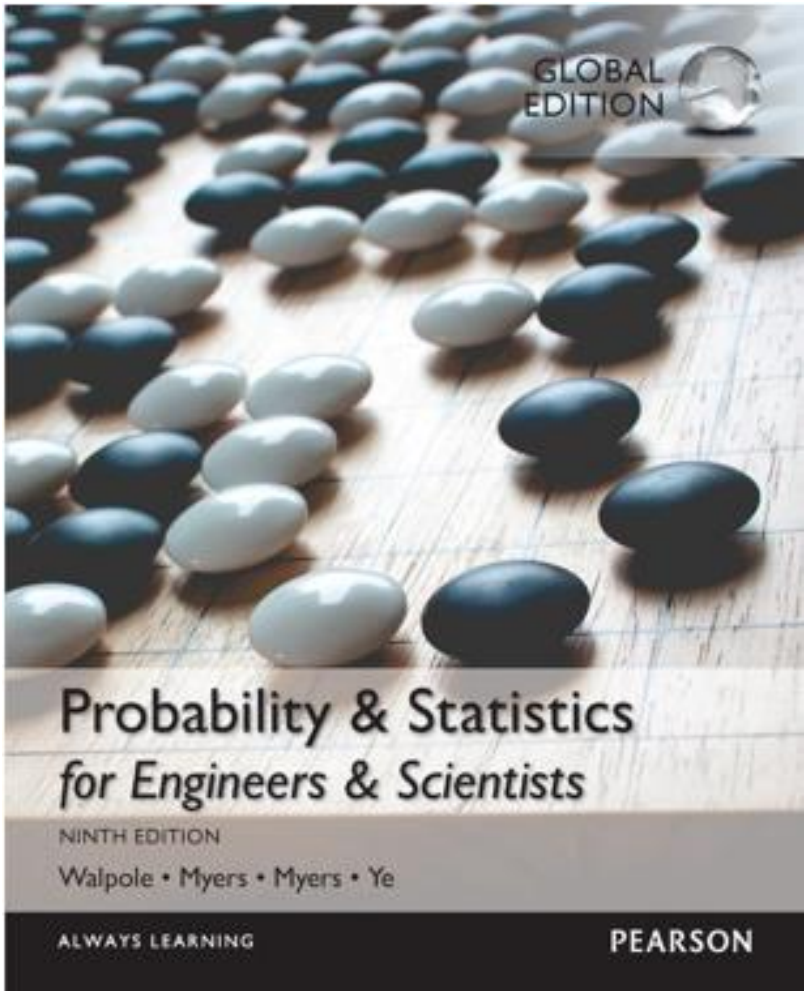
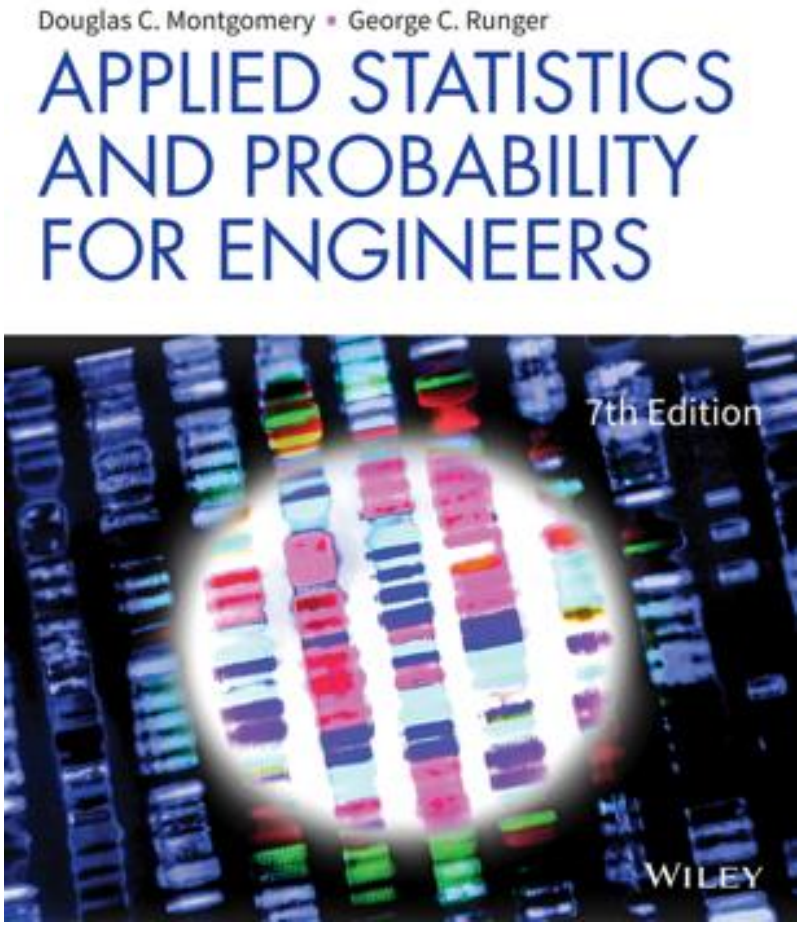
# Lectures References (1/2)




## Probability & Statistics for Engineers & Scientists

9th Edition

# Lectures References (2/2)




## Applied Statistics and Probability for Engineers

7th Edition

# Course Syllabus

- Chapter 1: Probability.
- Chapter 2: Random Variables.
- Chapter 3: Probability Distributions.
- Chapter 4: Descriptive Statistics.
- Chapter 5: Estimation (Confidence Interval).
- Chapter 6: Hypothesis Tests.

# Chapter 1: Probability

- Sample Space.
- Events.
- Counting Techniques.
- Probability of an Event.
- Additive Rules.
- Conditional Probability.
- Independence, and the Product Rule.
- Bayes' Rule.

# Sample Space (1/9)

## Random (Statistical) Experiment:

- An experiment <with known outcomes> whose outcome cannot be predicted with certainty, before the experiment is run.

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- An experiment <with known outcomes> whose outcome cannot be predicted with certainty, before the experiment is run.

*The roll of a dice*

*The toss of (flipping) a coin*

# Sample Space (2/9)

## Sample Space ($S$):

- Set of ALL possible outcomes of a random experiment.
- A sample space is discrete if it consists of a finite or countable infinite set of outcomes.
- A sample space is continuous if it contains an interval (either finite or infinite) of real numbers.

# Sample Space (3/9)

## Sample Space ($S$):

- Set of ALL possible outcomes of a random experiment.

*The roll of a dice*

# Sample Space (3/9)

## Sample Space ($S$):

- Set of ALL possible outcomes of a random experiment.

*The roll of a dice*

$$S = \{1,2,3,4,5,6\}$$

# Sample Space (3/9)

## Sample Space ($S$):

**Discrete**

- Set of ALL possible outcomes of a random experiment.

*The roll of a dice*

$$S = \{1,2,3,4,5,6\}$$

Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**.

# Sample Space (4/9)

## Sample Space ($S$):

- Set of ALL possible outcomes of a random experiment.

Flipping a coin

# Sample Space (4/9)

## Sample Space ($S$):

- Set of ALL possible outcomes of a random experiment.

Flipping a coin

$$S = \{Head, Tail\}$$
$$S = \{H, T\}$$

# Sample Space (5/9)

## Example1:

Find the sample space for the random experiments (flipping) a coin of two times?

# Sample Space (5/9)

## Example1:

Find the sample space for the random experiments (flipping) a coin of two times?

## Answer:

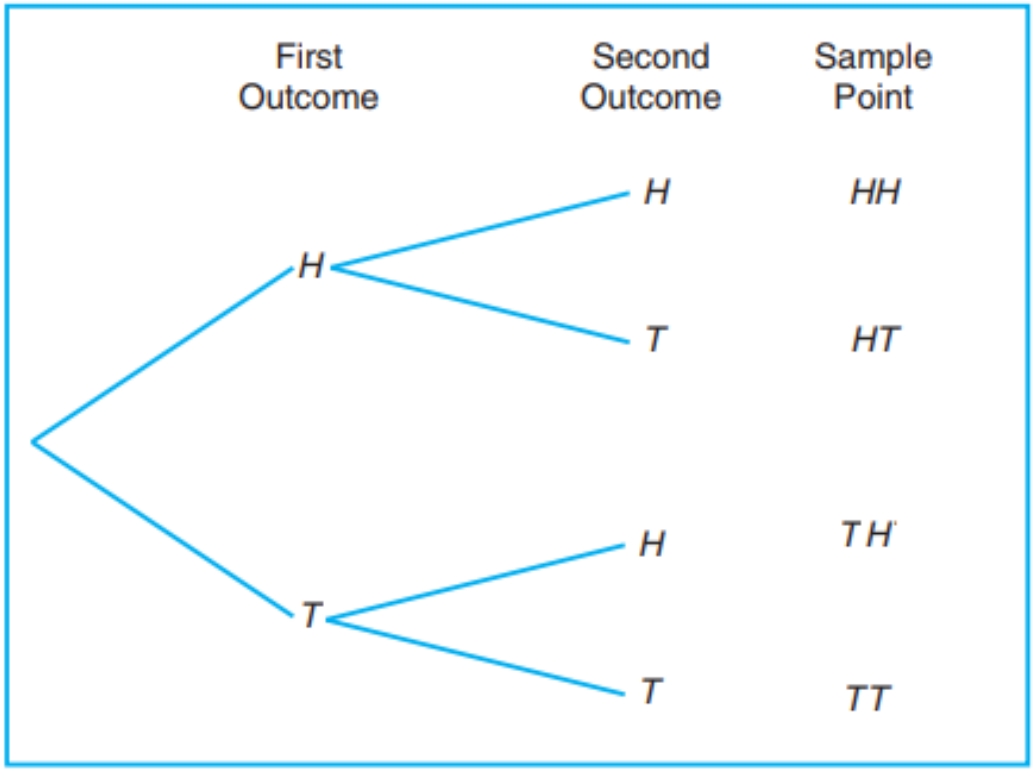
$S = \{HH, HT, TH, TT\}$

# Sample Space (6/9)

## Tree Diagrams:

Sample spaces can also be described graphically with *tree diagrams*.

$$S = \{HH, HT, TH, TT\}$$



# Sample Space (7/9)

## Example2:

An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once.

# Sample Space (7/9)

## Example2:

An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once.

## Answer:

$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

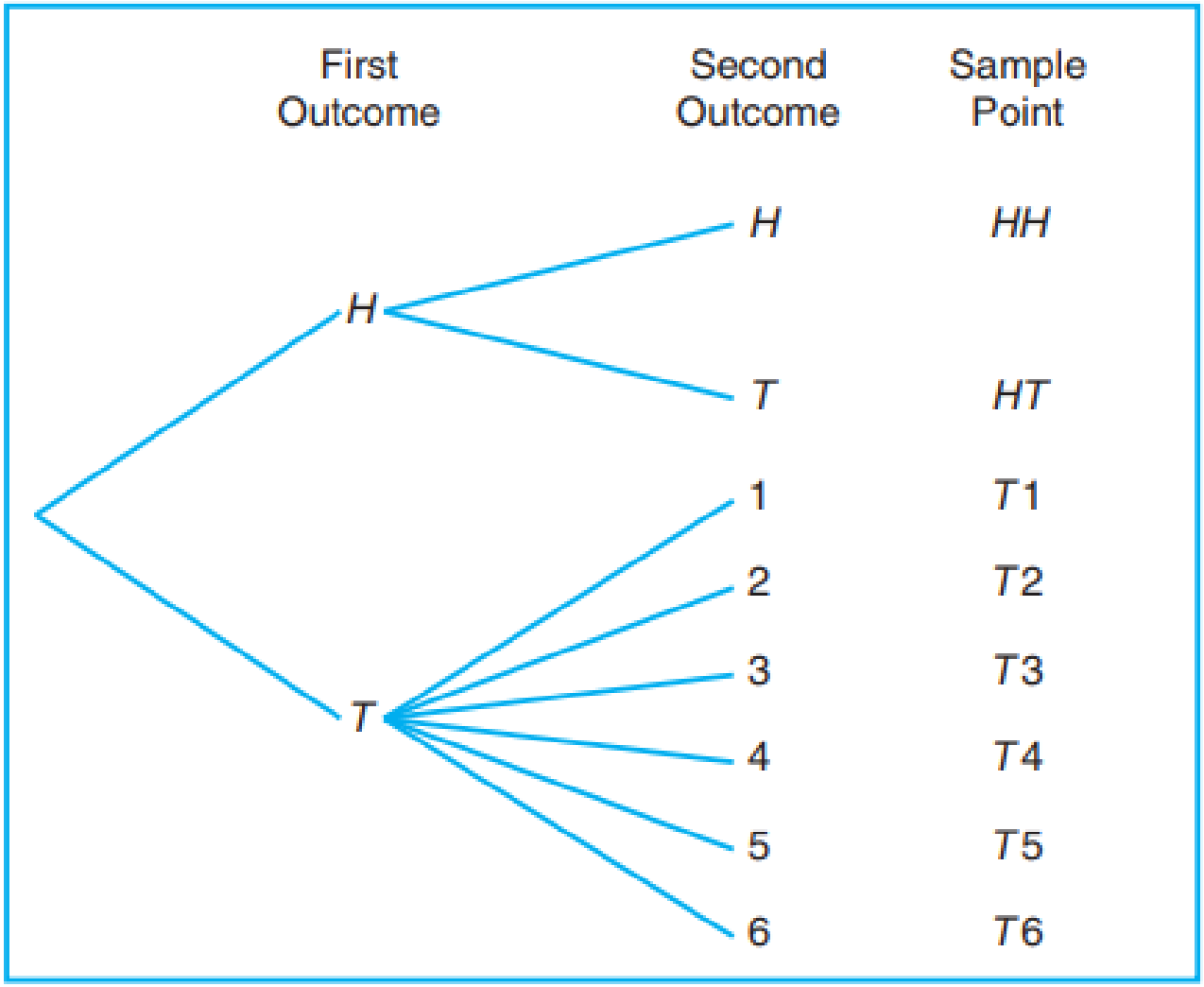
# Sample Space (8/9)

**Example2:**

$S =$

$\{HH, HT, T1, T2,$

$T3, T4, T5, T6\}$



# Sample Space (9/9)

## Example3:

**Continuous**

Consider an experiment that selects a cell phone camera and records the recycle time of a flash (the time taken to ready the camera for another flash).

$$\boldsymbol{S} = R^{+} = \{x | \, x > 0\}$$

If it is known that all recycle times are between 1.5 and 5 seconds, the sample space can be

$$\boldsymbol{S} = \{x | \, 1.5 < x < 5\}$$

# Events (1/19)

## Event ($E$):

- A result of *none* , *one* , or *more* outcomes in the sample space. An event is a subset of the sample space of a random experiment.

# Events (2/19)

## Event ($E$):

- A result of *none* , *one* , or *more* outcomes in the sample space. An event is a subset of the sample space of a random experiment.

*The roll of a dice*

$$S = \{1,2,3,4,5,6\}$$
$$E = \{2,4,6\}$$

**Even Numbers**

# Events (3/19)

## Example1:

A dice is rolled twice. What is the Event that the sum of the faces is greater than 7, given that the first outcome was a 4?

# Events (4/19)

## Example1:

A dice is rolled twice. What is the Event that the sum of the faces is greater than 7, given that the first outcome was a 4?

## Answer:

$$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26,$$

$$31, 32, 33, 34, 35, 36, \mathbf{41}, \mathbf{42}, \mathbf{43}, \mathbf{44}, \mathbf{45}, \mathbf{46},$$

$$51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$$

$$E = \{44, 45, 46\}$$

# Events (5/19)

We can also be interested in describing new events from combinations of existing events. Because events are subsets, we can use basic set operations such as *unions*, *intersections*, and *complements* to form other events of interest. Some of the basic set operations are summarized here in terms of events:

1. The union of two events is the event that consists of all outcomes that are contained in either of the two events. We denote the union as $E_1 \cup E_2$.

# Events (6/19)

We can also be interested in describing new events from combinations of existing events. Because events are subsets, we can use basic set operations such as *unions*, *intersections*, and *complements* to form other events of interest. Some of the basic set operations are summarized here in terms of events:

2. The intersection of two events is the event that consists of all outcomes that are contained in both of the two events. We denote the intersection as $E_1 \cap E_2$.

# Events (7/19)

We can also be interested in describing new events from combinations of existing events. Because events are subsets, we can use basic set operations such as *unions*, *intersections*, and *complements* to form other events of interest. Some of the basic set operations are summarized here in terms of events:

3. The complement of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the complement of the event $E$ as $E'$. The notation $E^c$ is also used in other literature to denote the complement.

# Events (8/19)

## Example2:

In the tossing of a die, we might let $A$ be the event that an even number occurs and $B$ the event that a number greater than 3 shows.

# Events (9/19)

## Example2:

In the tossing of a die, we might let $A$ be the event that an even number occurs and $B$ the event that a number greater than 3 shows.

Then the subsets $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$ are subsets of the same sample space $S = \{1, 2, 3, 4, 5, 6\}$.

# Events (10/19)

## Example2:

Then the subsets $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$ are subsets of the same sample space $S = \{1, 2, 3, 4, 5, 6\}$.

$A \cap B = \{4, 6\}$

$A \cup B = \{2, 4, 5, 6\}$

$A' = \{1, 3, 5\}$

$B' = \{1, 2, 3\}$

# Events (11/19)

## Mutually Exclusive, or Disjoint:

Two events $A$ and $B$ are *mutually exclusive*, or *disjoint*, if $A \cap B = \emptyset$, that is, if $A$ and $B$ have **no** elements in common.

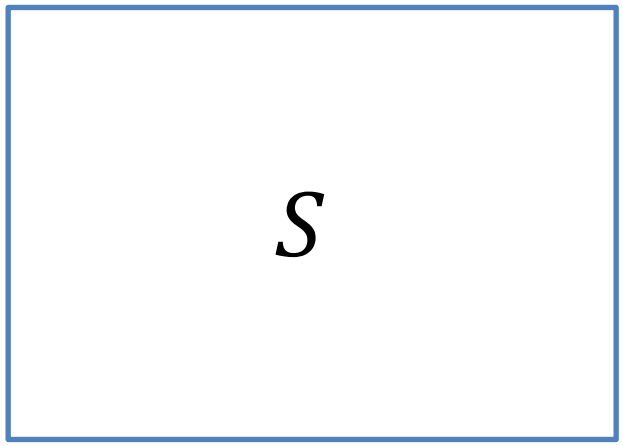
$A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$

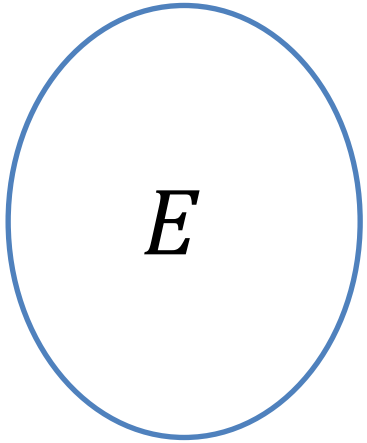
$A \cap B = \{\ \} = \emptyset$

# Events (12/19)

## Venn Diagrams:

Diagrams are often used to portray relationships between sets, and these diagrams are also used to describe relationships between events. We can use Venn diagrams to represent a sample space and events in a sample space.



# Events (13/19)

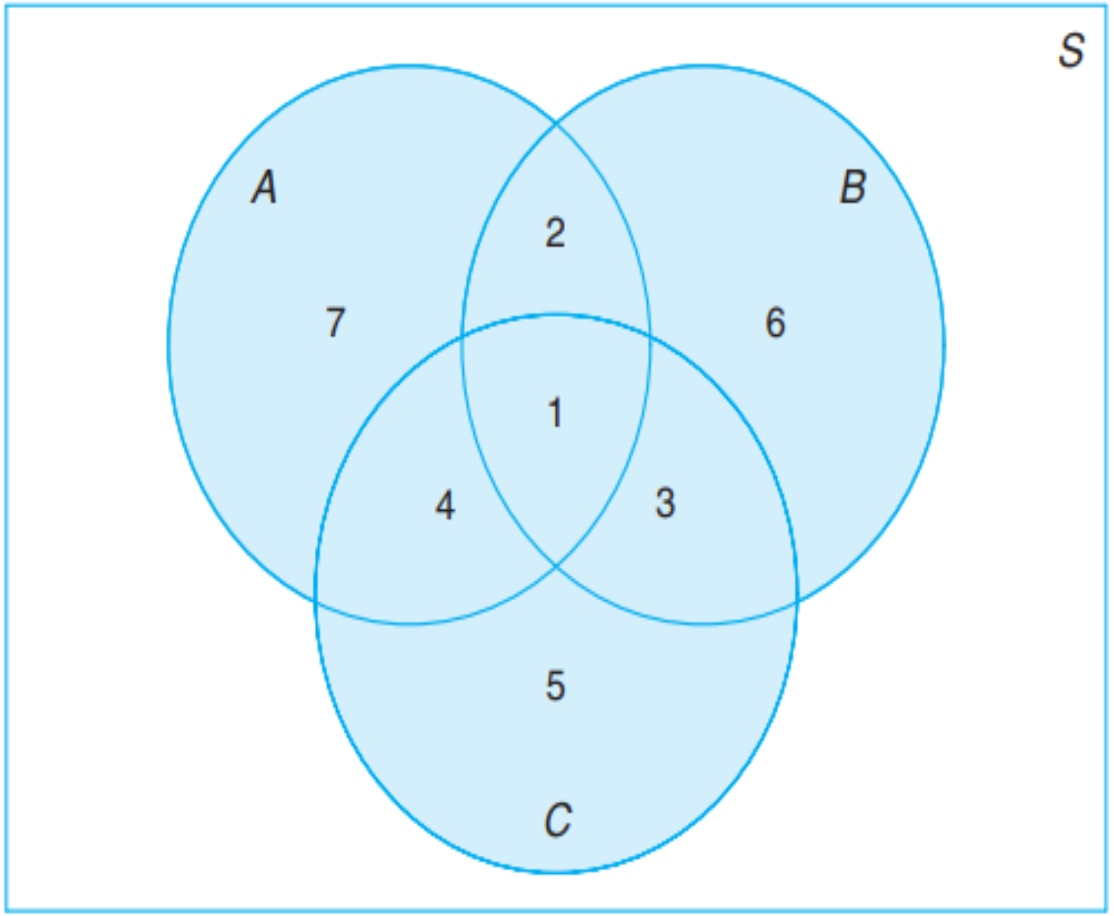
## Example1:

$S = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 2, 4, 7\}$

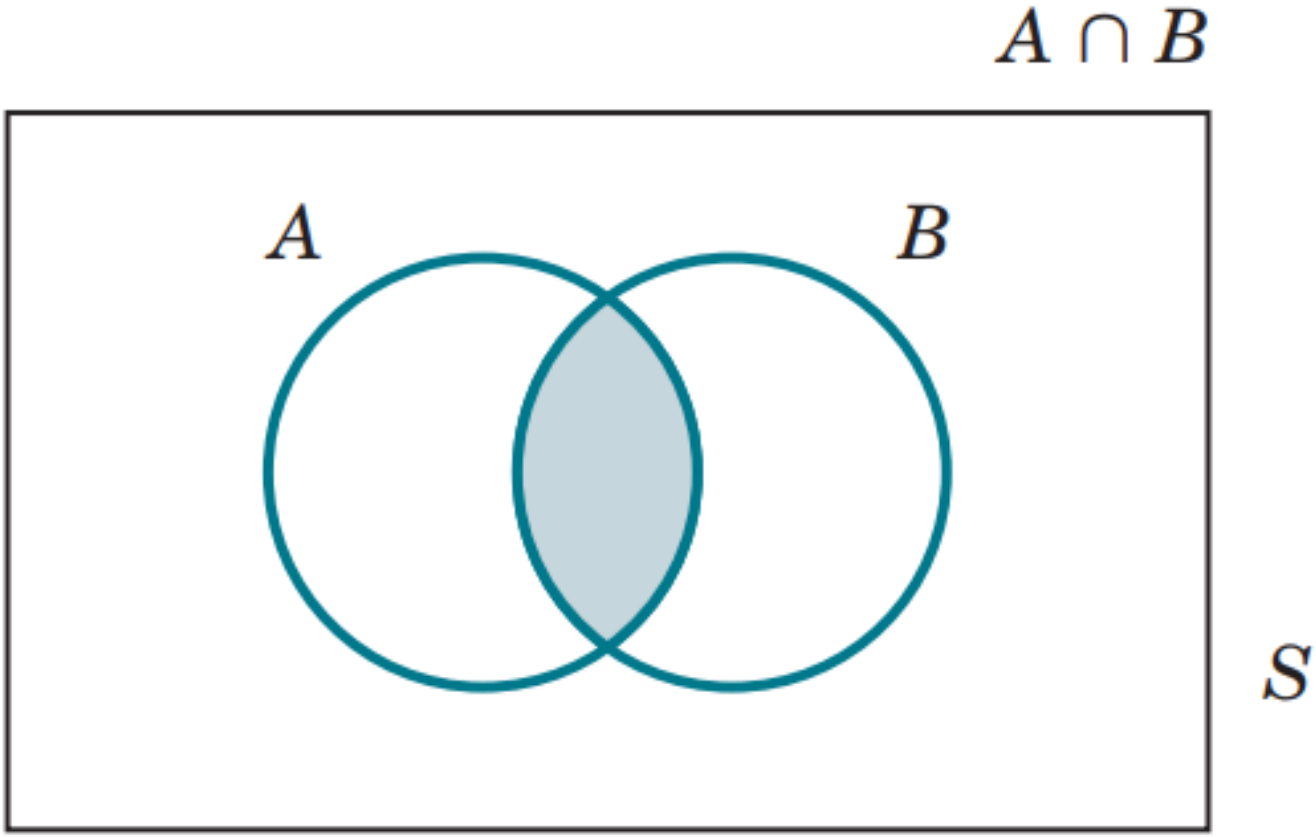
$B = \{1, 2, 3, 6\}$

$C = \{1, 3, 4, 5\}$



# Events (14/19)

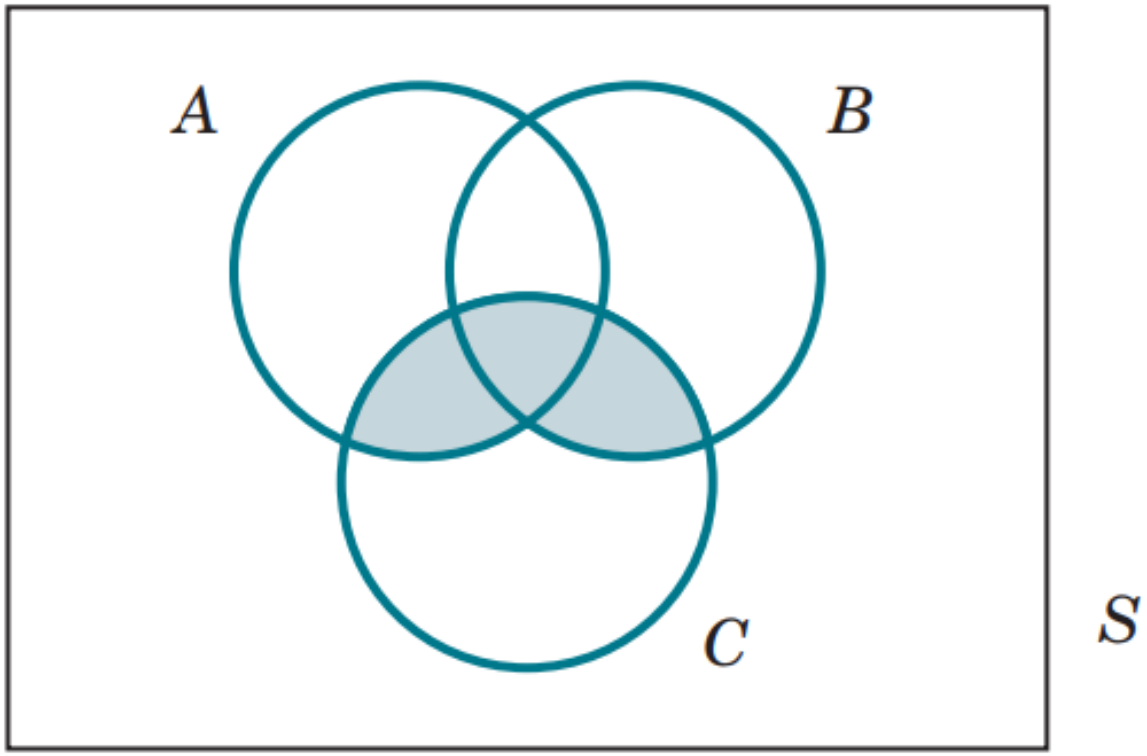
## Example2:



# Events (15/19)

## Example3:

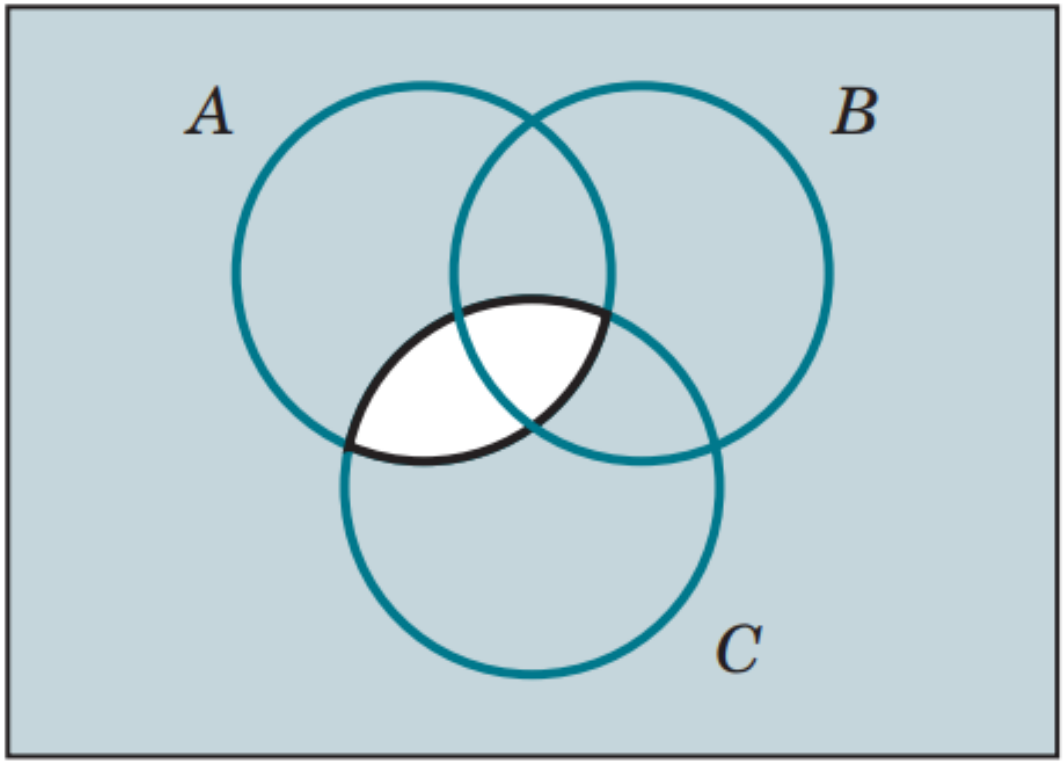
$(A \cup B) \cap C$



# Events (16/19)

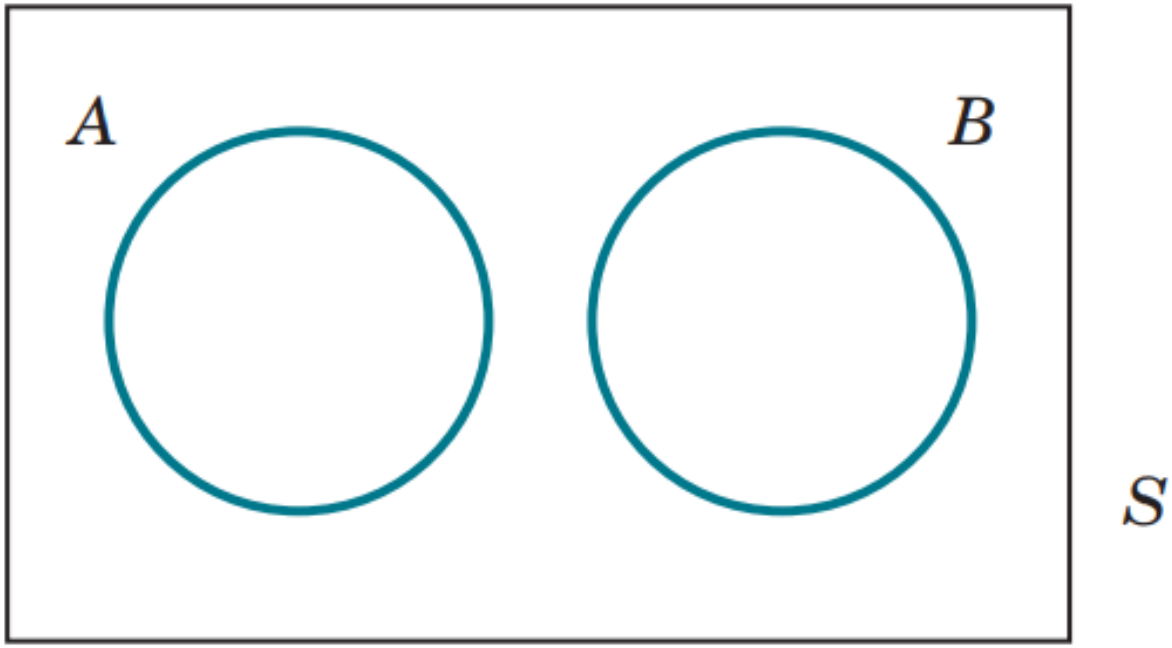
## Example4:

$(A \cap C)'$



# Events (17/19)

## Example5:




**Mutually exclusive events.**

# Events (18/19)

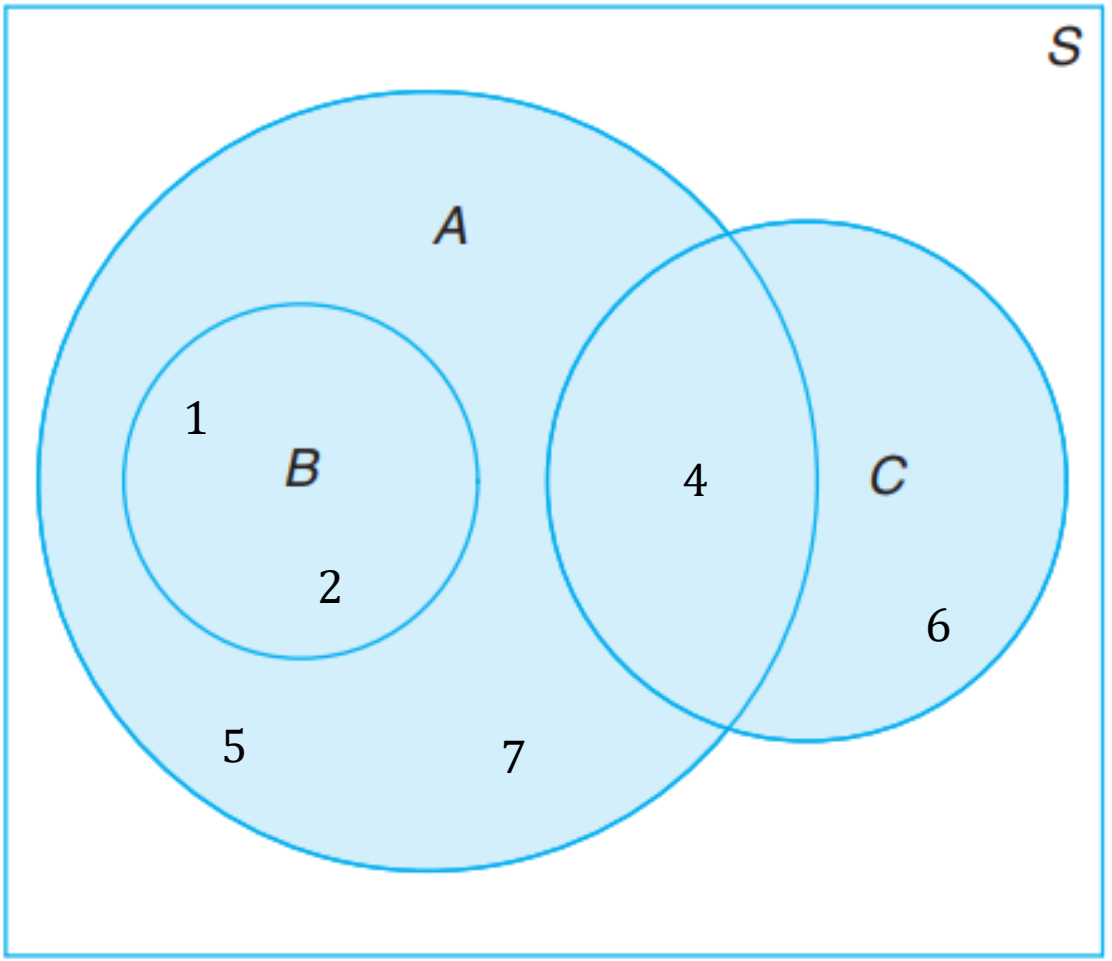
## Example6:

$S = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 2, 4, 5, 7\}$

$B = \{1, 2\}$

$C = \{4, 6\}$



# Events (19/19)

## Several Results:

1. $A \cap \phi = \phi.$
2. $A \cup \phi = A.$
3. $A \cap A' = \phi.$
4. $A \cup A' = S.$
5. $S' = \phi.$
6. $\phi' = S.$
7. $(A')' = A.$
8. $(A \cap B)' = A' \cup B'.$
9. $(A \cup B)' = A' \cap B'.$

# Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #1: https://www.youtube.com/watch?v=GmJJ2iZz08c&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=1&t=1s

# Thank You

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